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RESEARCH ARTICLE

LRS BIANCHI TYPE II COSMOLOGICAL MODELWITH STIFF FLUID AND VARYING Λ TERM

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- Deceleration Parameter
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ABSTRACT

In this paper, Einstein's Field equations for locally-Rotationally - Symmetric Bianchi type –II space time in the presence of stiff fluid matter and variable cosmological term are considered. To obtain the exact solution of Non – linear differential equations, we assume time-decaying cosmological term of the forms, $\Lambda = \alpha\ \dot{a}/a$, where a being average scale factor, By using a special law of variation for Hubble's parameter [Berman, M.S. (1983). Nuovo cimento, B 74,182], which yields a constant value of deceleration parameter.

A detail study of Physical parameters is carried out. The nature of singularities is also discussed.

INTRODUCTION

Bianchi cosmological models in general relativity provide a framework for investigation of the evolution of the universe. Our Present cosmology is based on the Friedman-Robertson-Walker (FRW) model and in this model, the universe is completely homogeneous and isotropic, which is in agreement with the observational evidences about the large scale structure of the universe. There are theoretical arguments [Chimento,L.P, Misner, C.W.] and recent observational data of the Cosmic Microwave Background (CMB) radiations which support the existence of an anisotropic phase that approaches an isotropic one by Land, K., et al. These experiments stimulates search for exact anisotropic solution of EFEs as cosmologically acceptable physical models for universe at least in its early stage of evolution of the universe.

Bianchi Type-II space time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of universe. Asseo, E., et al. emphasized the importance of Bianchi Type-II universe. Locally rotationally symmetric (LRS) Bianchi type-II space times have already been considered by a number of authors. Lorenz, D.et al. presented the exact solutions for LRS Bianchi II space time. Boutros, H., studied Bianchi type-II space time, with prefect fluid by a generating technique and also constructed LRS Bianchi type-II perfect fluid with an equation of state, which is function of time. Chakraborty S. had obtained solution of LRS Bianchi type-II with variable G and . Shanthi, K., et al., studied the same model in Barber's self creation theory of gravitation. Venkateswarlu, A., et al., found cosmological solutions for Bianchi-II stiff fluid models in electromagnetic field theory. Coley,A. et al, studied LRS Bianchi Type-II in to fluid Bianchi cosmologies. Singh, C.P. et al , have studied Bianchi type-II models with constant declaration parameter. Recently Pradhan et al. have investigated Bianchi type-II cosmological models with decay law of as where a is a scale factor. Tiwari,R. et al. have studied isotropic and anisotropic

cosmological models by taking where is the Hubble parameter, R is scale factor and m is a positive constant. The recent observations indicate that while particle physics prediction for is greater than this value by a factor of order 10120. This discrepancy is known as cosmological constant problem. The simplest way out of this problem is to consider a varying cosmological term, which decays from huge value at initial times to the small value observed in these days in an expanding universe Bertolami, O., Ozer, K., Freese, K..,et al Chen and Wu. Several phenomenological models have been suggested by considering as a function of time Corvalho, J.C. et al., Arbab A.I. Beesham, A. Vishwakarma R.G., Gasperini, M., Berman,M.S., Ozer,K. et al, Peebles and Ratra , Dussattar,A.B. et al, Garid et al, Pradhan et al. Of the special interest is the ansätz (where a is the scale factor of the Roberston-Walker metric) by Chen and Wu , which has been modified by several authors: Abdel-Rahman,A. , Corvalho,I. et al., Silveira,V. et al., Vishwakarma,R.G., However, not all vacumm decaying cosmological models predict acceleration. AI- Rawaf et al., and Overdin et al. proposed a cosmological model with a cosmological term of the form , where a is the scale factor of the universe and m is a constant. The recent observational evidences for an accelerated state of the present universe, obtained from distant Supernovae Ia, Riess, A., et al, gave strong support to search for alternative cosmologies. Thus the state of affairs has stimulated the interest. In more general models containing an extra component describing dark energy, and simultaneously accounting for the present accelerated stage of the universe.
In this paper, we consider EFEs for anisotropic LRS Bianchi type- II space time in the presence of stiff fluid for two types of cosmologies (power law cosmology and exponential cosmology) with time-decaying cosmological term of the forms, $\Lambda = \alpha\, \ddot{a}/a$, By using a special law of variation for Hubble's parameter proposed by Berman, M.S. to get an exact solutions EFEs. These solutions represents an anisotropic Bianchi type II stiff fluid cosmological model with negative constant deceleration parameter which corresponds to the accelerated state of the present universe.

THE METRIC AND FIELD EQUATIONS

We Consider the LRS (Locally Rotationally Symmetric) Metric for anisotropic Bianchi Type II Space-time in the form,

$$ds^2 = g_{ij}\theta^i\theta^j\ , \quad g_{ij} = \text{diag}\,(-1,1,1,1) \tag{1}$$

where, the Cartan bases θ^i are given by,

$$\theta^0 = dt,\ \ \theta^1 = S(t)\omega^1,\ \ \theta^2 = R(t)\omega^2\ ,\ \ \theta^3 = R(t)\,\omega^3 \tag{2}$$

where, R(t) and S(t) are the functions of cosmic time t. The time-dependent differential one forms ω^i are given by,

$$\omega^1 = dy + x dz,\ \ \omega^2 = dz,\ \ \omega^3 = dx \tag{3}$$

For metric (1), The Spatial average scale factor a is given by,

$$a^3 = (R^2 S) \tag{4}$$

The Volume scale factor V is given by,

$$V = a^3 \tag{5}$$

Also we define the average Hubble's parameter H as,

$$H = \frac{1}{3}(H_1 + H_2 + H_3) \tag{6}$$

where, $H_1 = H_2 = \frac{\dot{R}}{R}$, $H_3 = \frac{\dot{s}}{s}$, are the directional Hubble's factor in the directions of x,y,z respectively. Here and elsewhere dot denotes derivative with respect to cosmic time t.From equation (4)-(6), the average Hubble's parameter may be generalized in anisotropic cosmological model as,

$$H = \frac{1}{3}\frac{\dot{V}}{V} \quad \Rightarrow \quad \frac{\dot{a}}{a} = \frac{1}{3}\left(2\frac{\dot{R}}{R} + \frac{\dot{s}}{s}\right) \tag{7}$$

Einstein's field equations for LRS-Bianchi type-II space time,

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} + \Lambda g_{ij} = -8\Pi\, T_{ij} \tag{8}$$

where, R_{ij}, R_k^k, g_{ij} are Ricci tensor, Ricci Scalar, metric tensor and Λ is cosmological constant here, T_{ij} is the energy momentum tensor of the cosmic matter given by,

$$T_{ij} = (\rho + p)\, u_i u^j - p\, g_{ij} \tag{9}$$

where, ρ and p are energy density and pressure of the cosmic fluid and u_i is the fluid four velocity vector such that $u_i u^i = 1$

We take the equation of state (EoS) as,

$$p = \omega\rho \tag{10}$$

where, ω is a constant and $0 \leq \omega \leq 1$.

The metric (1) and the energy momentum tensor (9) co-moving co-ordinates, the field equation (8) gives,

$$2\frac{\dot{R}}{R}\frac{\dot{S}}{S} + \frac{\dot{R}^2}{R^2} - \frac{S^2}{4R^4} = 8\pi\rho + \Lambda \tag{11}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} - \frac{3S^2}{4R^4} = -8\pi p + \Lambda \tag{12}$$

$$\frac{\ddot{S}}{S} + \frac{\ddot{R}}{R} + \frac{\dot{R}}{R}\frac{\dot{S}}{S} + \frac{S^2}{4R^4} = -8\pi p + \Lambda \tag{13}$$

The usual energy conservation equation is given by,

$T_{i,j}^{j} = 0$, yields,

$$\dot{\rho} + (\rho + p)\left(2\frac{\dot{R}}{R} + \frac{\dot{s}}{s}\right) = -\frac{\dot{\Lambda}}{8\pi} \tag{14}$$

Einstein equations (11)-(13) are a coupled system of highly non-linear differential equations. In order to obtain an exact solution, we assume a form for a matter content or relation between the metric functions. The solutions to the field equations may also be generated by applying the law of variation for Hubble's parameter, initially proposed by Berman , M.S. for FRW models, which yields a constant value of the deceleration parameter. Berman et al, Johri V.B. et al, Singh,G.P. et al, Pradhan et al, Reddy et al., Adhav et al. Singh et al and others have studied cosmological models with constant deceleration parameter. We obtain an exact solution of the Einstein's field equations with the help of law of variation for

Hubble's parameter, which represents an anisotropic cosmological model with negative constant deceleration parameter.

SOLUTION OF THE FIELD EQUATIONS

LRS Bianchi type-II space time that also yields a constant value of deceleration parameter according to the proposed law, the variation of Hubble's parameter is given by[45],

$$H = \frac{k}{a^m} \tag{15}$$

where, ($k \geq 0$) and (m ≥ 0) are constants.

The spatial average scale factor a is given by,

$$a^3 = R^2 S \tag{16}$$

from equation (15) and (16), we get,

$$R^2 S = (mkt + k_1)^{3/m}, \quad m \neq 0 \tag{17}$$

$$R^2 S = {k_2}^3 e^{3kt}, \quad m = 0 \tag{18}$$

where, k_1 and k_2 are constants of integration.

The deceleration parameter q is defined as,

$$q = \frac{-a\ddot{a}}{\dot{a}^2} \tag{19}$$

Using equations (16) and (17) in equation (19),

$$q = \frac{m-1}{km}, \quad m \neq 0 \tag{20}$$

$$q = -1, \quad m = 0 \tag{21}$$

Equation (20) shows that, the law (15) gives a constant value of deceleration parameter (m≠0). Equation (21) shows that the law (15) gives a negative constant value of deceleration parameter (m=0).

The Proposed law (15), provides an alternative and straight forward approach to get the exact solutions of highly non-linear Einstein's field equations for Bianchi Models in a very simple manner. We consider the case, when the space time is filled with stiff matter (ω =1).

In case of Stiff-matter equation (11) and (13) gives,

$$\frac{\ddot{S}}{S} + \frac{\ddot{R}}{R} + \frac{3\dot{R}}{R}\frac{\dot{S}}{S} + \frac{\dot{R}^2}{R^2} = 2\Lambda \tag{22}$$

On integration, which gives,

$$R^2\dot{S} + R\dot{R}S = \int 2\Lambda\,(R^2 S)dt + h \tag{23}$$

where h is he constant of integration.

Now we solve the equation (23) for the scale factor R(t) and S(t) using equation (17) and (18) for all possible values of m in the following sections ;

3.1 Power Law Cosmology (m≠0)

Using equation (17) into equation (23) we get ,

$$R^2\dot{S} + R\dot{R}S = \int 2\Lambda\,(mkt + k_1)^{3/m}\,dt + \text{h} \tag{24}$$

Now we use the phenomenological decay law for Λ(t) of the form,

$$\Lambda = \frac{\alpha\dot{a}}{a} \tag{25}$$

By using the equation (4),(17) & (25) into equation (24), we get,

$$R^2\dot{S} + R\dot{R}S = \frac{2\alpha km}{3}\,(mkt + k_1)^{3/m} + \text{h} \tag{26}$$

which leads,

$$\text{RS} = \text{m}_1\,\exp\left[\frac{2\alpha km}{3}t + \frac{mh}{(m-3)}(mkt + k_1)^{m-3/m}\right] \tag{27}$$

where, ($m_1 > 0$) is a constant of integration and (m≠3).

Solving equation (17) and (27) we get,

$$\text{R(t)} = \frac{1}{\text{m1}}\,(mkt + k_1)^{3/m}\,\exp\left[\frac{-2\alpha km}{3}t - \frac{mh}{(m-3)}(mkt + k_1)^{m-3/m}\right] \tag{28}$$

$$\text{S(t)} = m_1^2(mkt + k_1)^{-3/m}\,\exp\left[\frac{4\alpha km}{3}t + \frac{2mh}{(m-3)}(mkt + k_1)^{m-3/m}\right] \tag{29}$$

Now, the directional Hubble's factors H_1, H_2 and H_3 in the directions of x, y, z are given by,

$$H_1 = H_2 = \frac{\dot{R}}{R} = \left[\frac{3k}{(mkt+k_1)} - \frac{2\alpha km}{3} - mhk(mkt + k_1)^{-3/m}\right] \tag{30}$$

$$H_3 = \frac{\dot{S}}{S} = \left[\frac{-3k}{(mkt+k_1)} + \frac{4\alpha km}{3} + 2mhk(mkt + k_1)^{-3/m}\right] \tag{31}$$

with average Hubble parameter H is given by,

$$\text{H} = \frac{1}{3}\left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) \tag{32}$$

$$\text{H} = \frac{k}{(mkt+k_1)} \tag{33}$$

The anisotropy parameter $\bar{A}$ is defined as,

$$\bar{A} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{34}$$

where $\Delta H_i = H_i - H$ [i=1,2,3]

using equation (30), (31) & (33) into equation (34), we get,

$$\bar{A} = 0 \tag{35}$$

The expansion scalar θ is given by,

$$\theta = 3\text{H} = \left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) \tag{36}$$

$$\theta = \frac{3k}{(mkt+k_1)} \tag{37}$$

The Shear scalar σ is defined as,

$$\sigma^2 = \sigma^{ij}\sigma_{ij} = \frac{1}{3}\left(\frac{\dot{R}}{R} - \frac{\dot{s}}{s}\right)^2 \quad (38)$$

$$\sigma = \frac{1}{\sqrt{3}}\left[\frac{6k}{(mkt+k_1)} - 2\alpha km - 3mhk(mkt+k_1)^{-3/m}\right] \quad (39)$$

Energy density ρ is defines as,

$$\rho = (mkt + k_1)^{-6/m}\exp\left[\frac{\alpha k}{8\pi(mkt+k_1)}\right] \quad (40)$$

Isotropic pressure p is defined as,

$$p = (mkt + k_1)^{-6/m}\exp\left[\frac{\alpha k}{8\pi(mkt+k_1)}\right] \quad (41)$$

We observe that, the metric (1) [for m≠0] with R(t) and S(t) given by equation (28) and (29) represents an exact stiff fluid LRS – Bianchi type-II Cosmological model with negative constant deceleration parameter and time decaying positive cosmological term Λ(t). We observe that, this is an accelerating model of the universe. And at t = $\frac{-k_1}{mk} = t_0$,the spatial volume is zero. One of the scale factors R(t) Vanishes while the other scale factor S(t) diverges at t = t_0, The model has the Cigar-type singularity at t_0. Directional Hubble's factors H_1 , H_2 and H_3 are infinite at t = t_0,. For metric (1), the anisotropic parameter is zero. At Hubble parameter (H), Expansion Scalar (θ), Energy density (ρ) & cosmic pressure (p) are infinite. Shear scalar (σ)is non-zero at t = t_0**.** Also it is observed that, At t → ∞,the ratio $\frac{\sigma}{\theta}$ does not tends to zero, which shows that the our model does not approaches isotropy at late time. The expansion scalar $\theta \to 0$ as t → ∞,indicates that the universe is expanding with increase of time and the rate of expansion decreases with increase of time.

3.2 Exponential Cosmology (m=0)

Using equation (18) into equation (23) we get,

$$R^2\dot{S} + R\dot{R}S = \int 2\Lambda \ (k_2{}^3 e^{3kt})\, dt + h \quad (42)$$

Now we use the phenomenological decay law for Λ(t) of the form,

$$\Lambda = \frac{\alpha\dot{a}}{a} \ , \quad (43)$$

By using the equation (4), (18) & (43) into equation (42) we get,

$$R^2\dot{S} + R\dot{R}S = \frac{2\alpha}{3} \ (k_2{}^3 e^{3kt}) + h \quad (44)$$

which leads,

$$RS = \exp\left\{\frac{2}{3}\alpha t - \frac{h}{3kk_2{}^3} e^{-3kt}\right\} \quad (45)$$

from equation (18) and (44) we get,

$$R(t) = k_2{}^3 \exp\left\{\left(3k - \frac{2}{3}\alpha\right)t + \frac{h}{3kk_2{}^3} e^{-3kt}\right\} \quad (46)$$

$$S(t) = {k_2}^3 \exp\left\{\left(\frac{4}{3}\alpha - 3k\right)t - \frac{2h}{3k{k_2}^3} e^{-3kt}\right\} \qquad (47)$$

Directional Hubble factors as,

$$H_1 = H_2 = \frac{\dot{R}}{R} = \left\{3k - \frac{2\alpha}{3k_2} - \frac{h}{{k_2}^3} e^{-3kt}\right\} \qquad (48)$$

$$H_3 = \frac{\dot{S}}{S} = \left\{-3k + \frac{4\alpha}{3k_2} + \frac{2h}{{k_2}^3} e^{-3kt}\right\} \qquad (49)$$

Hubble's parameter is,

$$H = k, \qquad (50)$$

Expansion Scalar is,

$$\theta = 3k, \qquad (51)$$

Anisotropic parameter is,

$$\bar{A} = 0, \qquad (52)$$

Energy density is,

$$\rho = \frac{1}{{k_2}^3} e^{-3kt} \qquad (53)$$

Cosmic Pressure is,

$$p = \frac{1}{{k_2}^3} e^{-3kt} \qquad (54)$$

Shear Scalar is,

$$\sigma = \frac{1}{\sqrt{3}}\left[6k - \frac{6k}{3k_2} - 3h\frac{1}{{k_2}^3} e^{-3kt}\right] \qquad (55)$$

5. Conclusion

In this Paper, we observed that EFEs for LRS Bianchi type II Space time in the presence of stiff build with time decaying cosmological term of the form $\Lambda \backsim \frac{\dot{a}}{a}$, The Solutions are obtained using special law of variation of Hubble's parameter proposed by Berman [45] for anisotropic models that yields a constant value of deceleration parameter. The law in equation (15) provides an alternative and easy approach to get the exact solutions of the highly Non-linear Einstein field equations for Bianchi type models in a very simple manner.

The nature of the singularities of the model has been clarified and explicit forms of scale factors have been obtained . It has been observed that the universe status expending at t = $\frac{-k}{mk}$ for m≠0 and t = 0 for m = 0. Therefore this model of the universe has a singular origin for m≠0 and Non-Singular origin for m = 0.

The deceleration parameter q = -1 for m = 0, indicating the value of q leads to $\frac{dH}{dt} = 0$, which implies the greatest value of Hubble's parameter and fastest rate of expansion of the universe. Also, it is observed that the ratio of shear scalar and expansion scalar is non-zero for all values of t. Hence the universe remains anisotropic throughout the evolution. Thus the model represents a shearing, non-rotating and expanding universe. Anisotropic parameter is zero for all value of m. Therefore, at the time of evolution of the universe anisotropy is constant. Thus our model for LRS Bianchi type II universe in the presence of stiff fluid is compatible with the recent observations.

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